

Massachusetts Mathematics League

Some Mathematical Terms and Ideas

Whose Understanding Will Be Assumed in Each Contest

- 1) If a diagram is given with a problem, it is not necessarily drawn to scale.
- 2) All answers must be in exact simplified form, unless otherwise specified.
- 3) Units of any answer, if required, must be the same as the units used in the statement of the problem, e.g. problem given in degrees means answer must be in degrees.
- 4) The word “**compute**” will always call for an exact answer in simplest form.
 - Fractions must be completely reduced. $\frac{9}{15} \rightarrow \frac{3}{5}$
 - All radicals must be simplified, i.e., square roots must be ‘square-free’, cube roots must be ‘cube-free’, etc. $\sqrt{12} \rightarrow 2\sqrt{3}$
 $\sqrt[3]{40} \rightarrow 2\sqrt[3]{5}$
 - Where possible, denominators must be rationalized. $\frac{6}{\sqrt{3}} \rightarrow 2\sqrt{3}$
(An answer such as $1/\pi$ would be left as is.) $\frac{6}{\sqrt[4]{48}} \rightarrow \sqrt[4]{27}$
- 5) If the base of a number is not specifically indicated, it is understood to be base 10.
- 6) Divisors (or factors) of a positive integer refer to positive integer divisors only. (Proper divisors of a positive integer are divisors that are less than the integer itself.)
- 7) Prime numbers are positive integers with exactly two different factors. Composite numbers have more than two different factors. Note: 1 is neither prime nor composite.
- 8) If a **ratio** $a : b$ is requested, then a colon (:) will be placed in the answer blank. The corresponding fraction $\frac{a}{b}$ must be in simplified form with a rationalized denominator, whenever possible. If $\frac{a}{b}$ represents an integer, then $b = 1$. The underlining will be considered a grouping symbol, e.g. $\underline{\sqrt{3}+1} : \underline{2-\sqrt{2}}$ will mean $(\sqrt{3}+1) : (2-\sqrt{2})$. Explicit parentheses are not required.
- 9) Lattice points are points both of whose coordinates are integers.

Advanced Stuff

- 10) a) The letter i will always be used for $\sqrt{-1}$. b) $180^\circ = \pi$ radians
- 11) The capital A in the expressions $\text{Arcsin } x$, $\text{Arccos } x$, and $\text{Arctan } x$ calls for the principal values of these inverse trigonometric functions. Their ranges are $-\frac{\pi}{2} \leq \text{Arcsin } x \leq \frac{\pi}{2}$, $0 \leq \text{Arccos } x \leq \pi$ and $-\frac{\pi}{2} < \text{Arctan } x < \frac{\pi}{2}$ respectively.
- 12) The product $n(n-1)(n-2) \cdots \cdot 2 \cdot 1$ is frequently written as $n!$ (and read as n factorial).
Note: As a special case, $0! = 1$.
- 13) The symbol $\binom{n}{r}$ is evaluated by the formula $\frac{n!}{r!(n-r)!}$. It denotes a combination of n things taken r at a time, i.e. a selection, where order is not important. (Alternate symbols $C(n, r)$ and ${}_n C_r$ are sometimes used.)
- 14) The symbol ${}_n P_r$ is evaluated by the formula $\frac{n!}{(n-r)!}$. It denotes a permutation of n things taken r at a time, i.e. an arrangement, where order is important. (The alternate symbol $P(n, r)$ is sometimes used.)
- 15) Unless otherwise specified, \log denotes the common logarithm, i.e. \log_{10} and \ln denotes the natural logarithm, i.e., \log_e , where $e \approx 2.718281\cdots$.
- 16) If a complex number in $a + bi$ form is required, then the question will always ask for all possible ordered pairs (a, b) .
- 17) If a complex number in $r\text{cis}\theta$ form is required, then the question will always ask for all possible ordered pairs (r, θ) .